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# Effects of Flow Dispersion on the Hollow Fiber Module Performance

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# NOTE

# Effects of Flow Dispersion on the Hollow Fiber Module Performance

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## Abstract

The flow rate dispersion in liquid/liquid dialysis through a hollow fiber module in general decreases the performance of hollow fibers. A mathematical model was developed to correlate the efficiencies and separation properties of a hollow fiber module with the flow rate dispersion. It was calculated that flow rate dispersion indeed decreases the efficiencies and separation properties of a hollow fiber module. This dispersion effect on the hollow fiber module may be minimized by increasing the flow rates for both the feed and dialysis solvent streams.

## INTRODUCTION

Liquid/liquid dialysis (*1, 2*) through selective membranes or hollow fibers might be useful in industry and biomedicine. Purification, separation, and the artificial kidney are examples of those applications. The development of a liquid/liquid dialysis process requires a detailed knowledge of the relationship among the parameters affecting the operation of dialysis. Among these parameters are the intrinsic properties of the membrane, the nature of dialysis solvents (*2*), the flow pattern of both feed and solvent streams, and the geometry of membranes. The effects of these membrane separation parameters using liquid/liquid dialysis under *idealized conditions* in the hollow fiber module were calculated and discussed before with the aid of a computer (*3*). These calculations were carried out for both single stage and multiple stages. It was then concluded that an efficient process using liquid/liquid dialysis requires a membrane with a high intrinsic permeability constant and a reasonable separation factor, as small an inner radius of hollow fiber as practical, and a dialysis solvent with an acceptable activity coefficient for the key species and a flow rate suitably coupled to that of the feed.

In the operation of liquid/liquid dialysis, both feed and solvent flow through the bore and shell sides of hollow fibers. As the feed solution flows through the shell side (or bore side) of a hollow fiber module, it may not flow uniformly around each fiber in the module. This is called flow dispersion. The effects of flow dispersion on the hollow fiber module performance, which include the efficiencies and separation properties of a hollow fiber module, will be discussed and calculated in this report. A mathematical model to correlate the hollow fiber module performance with the flow rate dispersion is developed.



## MODULE PERMEATION EQUATIONS

The module permeation equation for any permeating molecule or ion in liquid/liquid dialysis may be given by

$$C(F_0) = C_0 \sum_{i=1}^{N} \frac{f_i}{F_0} \exp\left(\frac{-P}{l}\frac{a_i}{f_i}\right) \tag{1}$$

Where $C(F_0)$ is the concentration for the permeating molecule or ion in the feed after dialysis in the module, and $C_0$ is the initial concentration in the feed before dialysis. $N$ is the total number of fibers, $f_i$ is the flow rate of the feed in the shell side for fiber $i$, $P/l$ is the intrinsic permeability, and $a_i$ is the surface area for fiber $i$. The boundary condition is

$$\sum_{i=1}^{N} f_i = F_0 \tag{2}$$

where $F_0$ is the flow rate for feed via the shell side (or bore side) of the hollow fiber module.

Equation (1) is only valid when the dialysis solvent is maintained in the hollow fiber module to give almost the maximum driving force for any permeating molecule or ion. This could be achieved in the cases of Donnan dialysis (*4, 5*), using dialysis solvents which could react or form the complex with permeating molecules (*6*), and using countercurrent dialysis with a high ratio of dialysis solvent versus feed flow rates (*3*).

When $N$ (number of hollow fibers) approaches a large number, one can use the integral equations to represent Eqs. (1) and (2):

$$C(F_0) = \frac{C_0}{F_0} \int_0^{\infty} NfP(f) \exp\left(\frac{-P}{l}\frac{a}{f}\right) df \tag{3}$$

$$\int_0^\infty NfP(f)\,df = F_0 \tag{4}$$

where $p(f)$ is called the distribution function of flow rate, and $a$ is the average cross-section area for each single fiber. The measured experimental permeability constant for hollow fiber module will be defined as

$$\left(\frac{P}{l}\right)_m = \frac{F_0}{A} \ln\,[C_0/C(F_0)] \tag{5}$$

where $A$ is the total surface area of hollow fibers in the module which is equal to $Na$, and the efficiency percentage for module is defined as

$$\alpha = \frac{(P/l)_m \times 100}{(P/l)} \tag{6}$$

Thus, depending upon the flow rate distribution in the hollow fiber, one can have various efficiencies for fiber modules as can be seen from the following cases.

*Case I*: Uniform flow rate:

$$P(f) = \delta(f - f_0) \tag{7}$$

The distribution function is a delta function. In this case the flow rate is uniform in all the fibers at a flow rate $f_0$. In other words, each fiber in the module sees the same flow rate of feed stream. One can calculate the permeability and efficiency for a hollow fiber module:

$$\left(\frac{P}{l}\right)_m = \left(\frac{P}{l}\right)$$

$$\alpha = 100\%$$

Thus we will have an ideal hollow fiber module in which every fiber in the hollow fiber module has the same efficiency as each individual fiber has if we can have a uniform flow rate for all the fibers in the module.

*Case II*: All or none flow rate:

$$p(f) = m\delta\left(f - \frac{1}{m}f_0\right) + (1 - m)\delta(f) \tag{8}$$

where

$$m \leq 1 \tag{9}$$

This is the case where $m$ fraction of total fibers see the feed solution flow and $(1 - m)$ fraction of fibers do not have any feed flow around them due to channeling or other effects. Then Eqs. (5) and (6) will become $(P/l)_m = m(P/l) = 100m\%$.

For example, if we have 10% of a hollow fiber through which the feed solution does not flow, then we have a 90% efficiency module.

*Case III*: A Gaussian-type flow rate distribution:

$$P(y) = \frac{2}{\pi}\frac{f_0}{\sigma}\frac{1}{y^3}\exp\left[(y - y_0)^2/2\sigma^2\right] \tag{10}$$

where

$$y = 1/f, \qquad y_0 = 1/f_0, \qquad \sigma = \sqrt{k}/f_0$$

where $k$ is constant ($k$ indicates the breadth of flow of distribution), and $f_0$ is the average flow rate per single fiber. Then

$$c(f) = C_0 \exp\left[-\frac{P}{l}\frac{a}{f_0}\left(1 - \frac{kaP}{2f_0 l}\right)\right] \tag{11}$$

$$\left(\frac{P}{l}\right)_m = \left(\frac{P}{l}\right)\left(1 - \frac{ka}{2f_0}\frac{P}{l}\right) \tag{12}$$

therefore

$$\alpha = \left(1 - \frac{ka}{2f_0}\frac{P}{l}\right) \times 100 \tag{13}$$

If there are two molecules or ions, $i$ and $j$, then the separation factor $(\mathrm{SF}_j{}^i)_m$ will become

$$(SF_j{}^i)_m = (SF_j{}^i)\frac{\left[1 - \frac{ka}{2f_0}\frac{P_i}{l}\right]}{\left[1 - \frac{ka}{2f_0}\frac{P_j}{l}\right]} \tag{14}$$

where

$$SF_j{}^i = P_i/P_j$$

Thus Eq. (14) indicates a decrease of separation factor that will occur due to the dispersion of flow rates.

Also, Eqs. (13) and (14) indicate that a high value of $(P/l)$ and the separation factor will require a high flow rate in order to achieve a high efficiency. The more dispersed the flow rate ($k$) is, the less the efficiency and separation factor will be. Thus one can determine the efficiency of a hollow fiber module from the measured permeation rate constant $(P/l)_m$ as a function of the flow rates for the feed solution.

## CONCLUSION AND DISCUSSION

We have discussed the effects of flow dispersion on a hollow fiber module performance. It was calculated that the flow dispersion in a hollow fiber module in general decreases performance (efficiencies and separation properties) of a hollow fiber module. These flow dispersion effects may be minimized by increasing the flow rates of the feed stream and dialysis solvent through a hollow fiber module.

There are some experiment evidences to indicate that the flow rate dispersion in a hollow fiber module is close to the Gaussian-type flow rate dispersion. The performance of a hollow fiber module as a function of the flow rates of feed stream appears to follow Eqs. (13) and (14).

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